

We note that a representation for the concentration in the form of the series (9) holds everywhere, with the exception of the neighborhood of the rear critical point $\theta \leq 0$ ($Pe^{-1/2}$) [1], where the thickness of the diffusional boundary layer $\delta = [j]^{-1}$ becomes infinitely great. Analogously to [7] it can be shown that the contribution of this region to the total diffusional flow to the drop is on the order of $O(Pe^{-1/2})$. Therefore, the calculation of succeeding terms of the series (9) leads to an improvement of formula (11) only in obtaining a solution for the concentration in the region of the rear critical point.

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EQUATIONS OF THERMOHYDROMECHANICS OF A TWO-PHASE POLYDISPERSE MEDIUM WITH PHASE TRANSITIONS HAVING A CONTINUOUS PARTICLE-SIZE DISTRIBUTION

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§1. We consider a heterogeneous mixture of two phases, in which the first phase is the carrier phase, while the second phase is present in the form of individual solid particles of different sizes, direct interaction between which can be neglected. We adopt the hypothesis of quasihomogeneity [1-3]: the distances at which the parameters of the flow vary significantly are much greater than the sizes of the particles themselves and the distances between them. At each point of the volume occupied by the liquid we can introduce the volumetric contents of the phases α_1 and the mean densities ρ_i ; here

$$\rho = \rho_1 + \rho_2, \quad \alpha_1 + \alpha_2 = 1, \quad \alpha_i \geq 0, \quad \rho_i = \rho_i^0 \alpha_i,$$

where the subscript 1 relates to the carrier phase, and 2 to the whole disperse phase; ρ_i is the density of the i -th component of the mixture. The dispersivity of the second phase is characterized by the function $f(r)$, so that $f(r)dr$ is the number of particles in unit volume of the mixture, whose dimensions (volumes) lie within the limits from r to $r + dr$. The density of the second phase is continuously distributed in the segment $[0, R]$, where R is the dimension (volume) of the largest particle. Consequently, we can write

$$\alpha_2 = \int_0^R f(r) r dr, \quad \rho_2 = \int_0^R \rho_2^0 f(r) r dr,$$

where ρ_2^0 is the true density of the disperse phase. We set $f(0) = f(R) = 0$. It is postulated that there are sufficient particles of all sizes so that it can be assumed that the carrier phase and any given set of particles (whose sizes lie in the segment r', r'' , where r' and r'' are any given values from the set $[0, R]$) are continua, filling exactly the same volume. The carrier phase is described by a model of a viscous liquid. Here, as the tensors of the surface forces σ_i^{kl} and the tensors of the viscous stresses τ_i^{kl} we take [1, 3]

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$$\sigma_1^{kl} = -p_1 \delta^{kl} + \tau_1^{kl}, \sigma_2^{kl} = 0, \tau_1^{kl} = \lambda_1 \nabla v_1 + 2\mu_1 e_1^{kl},$$

where δ^{kl} is a Kronecker symbol; λ_1, μ_1 are the viscosity coefficients; e_1^{kl} is the tensor of the deformation rates of the carrier phase.

§2. We introduce a phase space with the coordinates x^1, x^2, x^3, r , where x^1, x^2, x^3 are the coordinates of ordinary Euclidean space; r is the volume of a particle. In the phase space, we isolate a fixed volume V^* bounded by the surface S^* . Then the equation of the conservation of mass for the disperse phase can be written in the form

$$\int_{V^*} \frac{\partial}{\partial t} (\rho_2^0 f r) dV^* = - \int_{S^*} \rho_2^0 f r v^n dS^* + \int_{V^*} \rho_2^0 f (\lambda - \mu) dV^* \quad (2.1)$$

$$(dV^* = dV dr = dx^1 dx^2 dx^3 dr).$$

The first term in the right-hand part of Eq. (2.1) characterizes the influx of mass through the surface S^* , where \mathbf{v} is a vector, whose coordinates are $dx^1/dt, dx^2/dt, dx^3/dt, dr/dt$; the second term characterizes the change in the mass of substance in the volume V^* due directly to phase transitions. The phase transitions can be arbitrarily divided into two reactions: growth of the particles (for example, the growth of crystals with crystallization) with the volumetric rate λ and the dissolution of particles with the volumetric rate μ , so that $dr/dt = \lambda - \mu = \eta$, where η is the observed rate of change in the volume of particles ($\eta \geq 0, \mu \geq 0$).

The equations of the conservation of mass of the carrier phase and the mass of the salt in the solution in the volume V of ordinary Euclidean space with the coordinates x^1, x^2, x^3 , bounded by the surface S , have the form

$$\int_V \frac{\partial \rho_1}{\partial t} dV = - \int_S \rho_1 v_1^n dS - \int_V \int_0^R (\lambda - \mu) \rho_2^0 f dr dV; \quad (2.2)$$

$$\int_V \frac{\partial (\rho_1 c)}{\partial t} dV = - \int_S \rho_1 c v_1^n dS - \int_V m \int_0^R (\lambda - \mu) \rho_2^0 f dr dV, \quad (2.3)$$

where \mathbf{v}_1 is the velocity of the carrier phase; c is the mass concentration of salt in solution; m is the ratio of the molecular weights of the anhydrous salt and the crystal hydrate.

The equations of conservation of momentum for the disperse and carrier phases have the form

$$\int_{V^*} \frac{\partial}{\partial t} (\rho_2^0 f \mathbf{v}_2) dV^* = - \int_{S^*} \rho_2^0 f r \mathbf{v}_2 v^n dS^* + \int_{S^*} \sigma_2^n dS^* + \int_{V^*} \rho_2^0 f \mathbf{F}_2 dV^* +$$

$$+ \int_{V^*} f r \rho_2^0 \mathbf{r}_{(12)} dV^* + \int_{V^*} [\rho_2^0 \lambda f \mathbf{v}_{(12)} - \rho_2^0 \mu f \mathbf{v}_{(21)}] dV^*; \quad (2.4)$$

$$\int_V \frac{\partial}{\partial t} (\rho_1 \mathbf{v}_1) dV = - \int_S \rho_1 \mathbf{v}_1 v_1^n dS + \int_S \sigma_1^n dS + \int_V \rho_1 \mathbf{F}_1 dV +$$

$$+ \int_V \int_0^R \rho_2^0 f \mathbf{r}_{(12)} dr dV - \int_V \left[\int_0^R (\rho_2^0 \lambda \mathbf{v}_{(12)} - \rho_2^0 \mu \mathbf{v}_{(21)}) dr \right] dV. \quad (2.5)$$

The first terms in the right-hand parts of Eqs. (2.4), (2.5) give the influx of momentum of the corresponding phase through the surface S^* and S ; $\mathbf{v}_2(r)$ is the velocity of particles of the dimension r ; the second and third terms are the actions of the external surface and mass forces, characterized, respectively, by the tensors $\sigma_1^{kl}, \sigma_2^{kl}$, and the vectors $\mathbf{F}_1, \mathbf{F}_2$; here $\sigma_2^{kl} = 0$; the last term characterizes the change in the momentum of the corresponding phase due to phase transitions; $\mathbf{v}_{1,2}(r)$ is the velocity of the carrier phase at the contact surface of the phases; $\mathbf{v}_{2,1}(r)$ is the velocity of particles of size r at the phase-contact surface. The force of the interaction between the carrier phase and the inclusions is repre-

sented in the form [1]

$$\int_0^R \rho_2^0 f r \mathbf{r}_{(12)} dr = - \int_0^R r f \nabla p_1 dr + \int_0^R \rho_2^0 f r \mathbf{f}_{(12)} dr$$

$$\left(\int_0^R \rho_2^0 f r \mathbf{r}_{(12)} dr = - \int_0^R \rho_2^0 f r \mathbf{r}_{(21)} dr \right),$$

here the first term on the right is connected with the action of the field of the pressure on the inclusions (the Archimedes force), while the second term characterizes the force interaction of the phases and is due to three effects [1]: $\mathbf{f}_{(12)} = \mathbf{f}_f + \mathbf{f}_m + \mathbf{f}_r$, where \mathbf{f}_f is the component arising due to viscous forces with interaction between the phases; \mathbf{f}_m is the force connected with the effect of the "added mass"; \mathbf{f}_r is the Magnus or Zhukovskii force.

Applying the Gauss-Ostrogradskii theorem to Eqs. (2.1)-(2.5), we obtain a description of the system in the form of differential equations

$$\frac{\partial f}{\partial t} + \nabla (f v_2) + \frac{\partial}{\partial r} (f \eta) = 0;$$

$$\frac{\partial \rho_1}{\partial t} + \nabla (\rho_1 v_1) = - \int_0^R \rho_2^0 f \eta dr; \quad \rho_1 \frac{\partial c}{\partial t} = (c - m) \int_0^R \rho_2^0 f \eta dr; \quad (2.6)$$

$$g \frac{D_2 v_2}{Dt} = - f r \nabla p_1 + g \mathbf{f}_{(12)} + g \mathbf{F}_2 + \rho_2^0 f \lambda (\mathbf{v}_{(12)} - \mathbf{v}_2) - \rho_2^0 f \mu (\mathbf{v}_{(21)} - \mathbf{v}_2);$$

$$\rho_1 \frac{d_1 v_1}{dt} = - \alpha_1 \nabla p_1 + \nabla^k v_1^k - \int_0^R \rho_2^0 f r \mathbf{f}_{(12)} dr + \rho_1 \mathbf{F}_1 - \int_0^R \rho_2^0 f [\lambda (\mathbf{v}_{(12)} - \mathbf{v}_1) - \mu (\mathbf{v}_{(21)} - \mathbf{v}_1)] dr, \quad (2.7)$$

where

$$g = \rho_2^0 r f; \quad d_1/dt = \partial/\partial t + v_1^k (\partial/\partial x^k);$$

$$D_2/Dt = \partial/\partial t + v_2^k (\partial/\partial x^k) + \eta (\partial/\partial r).$$

§3. We adopt the hypothesis of the main thermodynamic characteristics with respect to the masses of the phases. Analogously to [1, 4], we introduce the surface component of the internal energy of the mixture, taking account of the surface (in accordance with Gibbs). Then we can write

$$\rho u = \rho_1 u_1 + \int_0^R (\rho_2^0 f r u_2 + f u_\sigma^1) dr;$$

$$\rho K = \frac{\rho_1 v_1^2}{2} + \int_0^R \rho_2^0 r f \frac{v_2^2}{2} dr,$$

where $\rho = \rho_1 + \int_0^R \rho_2^0 f r dr$; u , u_1 are the specific internal energies of the whole mixture and the carrier phase, respectively; $u_2(r)$ is the specific internal energy of particles of size r ; $u_\sigma^1 = 4\pi a^2 u_\sigma$; u_σ^1 and u_σ are the surface energies, attributed, respectively, to one inclusion and to a unit of surface; a is the radius of the particles; K is the kinetic energy of the mixture, determined without taking account of small-scale effects (which will be neglected in the present work).

Following reasoning, and carrying out computations, analogous to those given in detail in [1, 3], we obtain differential equations for the internal energies of the carrier phase, of particles of size r , and of the phase interface

$$\rho_1 \frac{d_1 u_1}{dt} = \int_0^R \{ -\rho_2^0 f \lambda x_{1(12)}(r) - \rho_2^0 f \mu x_{1(21)}(r) - q_{1\sigma}(r) \} dr - \nabla \mathbf{q}_1 + \rho_1 Q_1 +$$

$$+ \tau_1^{hl} e_1^{hl} + \int_0^R \kappa_1 g \mathbf{f}_{(12)} (\mathbf{v}_1 - \mathbf{v}_2) dr + \int_0^R \left\{ -\rho_2^0 f \lambda \frac{(\mathbf{v}_{(12)} - \mathbf{v}_1)^2}{2} + \rho_2^0 f \mu \frac{(\mathbf{v}_{(21)} - \mathbf{v}_1)^2}{2} \right\} dr; \quad (3.1)$$

$$g \frac{D_2 u_2}{Dt} = -\rho_2^0 f \lambda x_{2(12)}(r) - \rho_2^0 f \mu x_{2(21)}(r) - q_{2\sigma}(r) + g Q_2 + \kappa_2 g \mathbf{f}_{(12)} (\mathbf{v}_1 - \mathbf{v}_2) +$$

$$+ \rho_2^0 f \lambda \frac{(\mathbf{v}_{(12)} - \mathbf{v}_2)^2}{2} - \rho_2^0 f \mu \frac{(\mathbf{v}_{(21)} - \mathbf{v}_2)^2}{2}; \quad (3.2)$$

$$f \frac{D_2 u_\sigma^1}{Dt} = -\rho_2^0 f \lambda x_{\sigma(12)}(r) - \rho_2^0 f \mu x_{\sigma(21)}(r) +$$

$$+ q_{1\sigma}(r) + q_{2\sigma}(r) + f \lambda \frac{2\sigma_S}{a} - f \mu \frac{2\sigma_S}{a}; \quad (3.3)$$

where $x_{i(12)}(r)$, $x_{i(21)}(r)$ are the heat fluxes from the i -th phase to the substance undergoing the transformation from the 1-st phase to particles with the size r , and from particles with the size r to the 1-st phase, respectively ($i = 1, 2, \sigma$); $q_{2\sigma}(r)$, $q_{1\sigma}(r)$ are the heat fluxes, referred to unit volume of the mixture, from particles of size r to the phase interface, and from the carrier phase to the phase interface, respectively; \mathbf{q}_1 is the external heat flux (thermal conductivity); $\mathbf{q}_1 = -\lambda_2 \nabla T_1$; λ_2 is the thermal-conductivity coefficient of the material of the 1-st phase; gQ_2 , $\rho_1 Q_1$ are the powers of the volumetric sources of heat; the coefficient $\kappa_i(r)$ indicates the fraction of the kinetic energy of the mixture due to the force interaction between the carrier phase and particles of size r , going over directly into the internal energy of the i -th phase [$\kappa_1(r) + \kappa_2(r) = 1$]; σ_S is the surface tension of the phase interface.

We separate out a fixed volume V , bounded by the surface S . By analogy with [2], we define the concept of the substantial derivative of the total energy of the mixture, representing the change in the total energy going to a unit volume of the mixture fixed in space, after subtraction of the change connected with the influx of mass through the boundaries of the isolated volume:

$$\int_V \frac{\partial}{\partial t} (\rho E) dV = \int_S \left\{ \rho_1 \left(u_1 + \frac{v_1^2}{2} \right) v_1^n + \right.$$

$$\left. + \int_0^R \left[g \left(u_2 + \frac{v_2^2}{2} \right) v_2^n + f u_\sigma^1 v_2^n \right] dr \right\} dS + \rho \frac{dE}{dt}. \quad (3.4)$$

After application of the Gauss-Ostrogradskii formula to (3.4), we obtain

$$\rho \frac{dE}{dt} = \rho_1 \frac{d_1}{dt} \left(u_1 + \frac{v_1^2}{2} \right) + \int_0^R \left\{ g \frac{d_2}{dt} \left(u_2 + \frac{v_2^2}{2} \right) + f \frac{d_2 u_\sigma^1}{dt} \right\} dr +$$

$$+ \int_0^R \left\{ \left[u_\sigma^1 \left(-\frac{\partial}{\partial r} (f\lambda) + \frac{\partial}{\partial r} (f\mu) \right) \right] + \left[(-\rho_2^0 f \lambda + \rho_2^0 f \mu) \left(u_1 + \frac{v_1^2}{2} \right) \right] + \right.$$

$$\left. + \left[\rho_2^0 r \left(-\frac{\partial}{\partial r} (f\lambda) + \frac{\partial}{\partial r} (f\mu) \right) \left(u_2 + \frac{v_2^2}{2} \right) \right] \right\} dr. \quad (3.5)$$

Starting from the determination (3.5) of the equations of motion (2.6), (2.7) and the equations for the components of the energy of the mixture (3.1)-(3.3), we obtain an explicit expression for the substantial derivative of the total energy of the mixture

$$\begin{aligned}
\rho \frac{dE}{dt} = & \int_0^R -\rho_2^0 f \lambda (u_1 - u_2 + x_{2(12)}(r) + x_{1(12)}(r) + \\
& + x_{\sigma(12)}(r) + \frac{p_1}{\rho_1^0} - \frac{p_2}{\rho_2^0} - \frac{2\sigma_S}{\rho_2^0 a}) dr + \int_0^R \rho_2^0 f \mu [u_1 - u_2 - \\
& - (x_{1(21)}(r) + x_{2(21)}(r) + x_{\sigma(21)}(r)) + \frac{p_1}{\rho_1^0} - \frac{p_2}{\rho_2^0} - \frac{2\sigma_S}{\rho_2^0 a}] dr - \\
& - \nabla (\alpha_1 p_1 v_1) - \int_0^R \nabla \left(\frac{g}{\rho_2^0} p_1 v_2 \right) dr + \nabla^h \tau_1^h v_1^l - \\
& - \nabla q_1 + \rho_1 F_1 v_1 + \int_0^R g F_2 v_2 dr + \rho_1 Q_1 + \int_0^R g Q_2 dr - \\
& - \int_0^R \rho_2^0 \frac{\partial}{\partial r} \left[f \lambda r \left(u_2 + \frac{v_2^2}{2} \right) \right] dr + \int_0^R \rho_2^0 \frac{\partial}{\partial r} \left[f \mu r \left(u_2 + \frac{v_2^2}{2} \right) \right] dr + \int_0^R \left\{ -\frac{\partial}{\partial r} (f \lambda u_0^1) + \frac{\partial}{\partial r} (f \mu u_0^1) \right\} dr. \quad (3.6)
\end{aligned}$$

By virtue of the definition of f , the last three terms are equal to zero. In addition, from the sense of dE/dt it follows that the change in the total energy of the mixture is determined only by the external action. Therefore, the expressions in the right-hand part of Eq. (3.6) (the type of sources of energy due to phase transitions) should be equal to zero. Introducing the enthalpy of the phases ($i_i = u_i + p_i/\rho_i^0$, $i = 1, 2$), we obtain

$$\begin{aligned}
x_{1(12)} + x_{2(12)} + x_{\sigma(12)} &= i_2 - i_1 + \frac{2\sigma_S}{\rho_2^0 a}, \\
x_{1(21)} + x_{2(21)} + x_{\sigma(21)} &= i_1 - i_2 - \frac{2\sigma_S}{\rho_2^0 a}.
\end{aligned}$$

Quantification of the model demands determination of the energies $x_{k(ij)}$. These supplementary relationships will be postulated as

$$\begin{aligned}
x_{1(12)} = x_{2(12)} = x_{1(21)} = x_{2(21)} &= 0, \\
x_{\sigma(12)} = -x_{\sigma(21)} &= i_2 - i_1 + 2\sigma_S/\rho_2^0 a.
\end{aligned} \quad (3.7)$$

In accordance with the relationship (3.7), the role of a source or sink of heat, required for the phase transitions, is played by the surface phase.

§4. Crystallization is generally directly considered as a process taking place in two consecutive steps [5]: mass transfer from the carrier phase to the growing face of the crystal, and the crystallization process proper, i.e., incorporation of the structure particles of the dissolved substance in the crystal lattice. The first stage can be described using the equation of external mass transfer

$$\rho_2^0 \dot{\eta} = \beta F_s (c_s^* - c^*), \quad (4.1)$$

where c , c^* is the concentration of the substance in the main mass of the solution and at the surface of the crystal; β is the mass-transfer coefficient; F_s is the surface of the crystal.

The mass, arriving at the boundary of the crystal, must enter into it due to a reaction at the surface, i.e.,

$$\rho_2^0 \dot{\eta} = \beta_c F_s (c_s^* - c_e)^n, \quad (4.2)$$

where c_e is the equilibrium concentration, corresponding to the temperature at the surface of the crystal $T_\sigma(r)$; $\beta_c [T_2(r), T_\sigma(r)]$ is the rate constant of the crystallization; n is the order of the kinetic "reaction."

If c^* can be determined from Eqs. (4.1), (4.2), then, substituting c^* into Eq. (4.1), the growth rate of the crystal can be determined. For example, if $n = 1$ (in the case of a "reaction" of first order), then

$$\rho_2^0 \eta = \frac{\beta \beta_c}{\beta + \beta_c} F_s (c - c_e).$$

Analogously to [1], we take $\kappa_1 = 1$, $\kappa_2 = 0$; i.e., we shall postulate that the fraction of the kinetic energy due to the force interaction between the carrier and disperse phases goes over directly into the internal energy of the carrier phase. It can be assumed that, at the phase-contact surface, the relationship is satisfied

$$v_{(21)}(r) = v_{(12)}(r) = v_2(r). \quad (4.3)$$

It is important to note that, with the relationships adopted (3.7), (4.3), there is no further need for the introduction of arbitrary growth rates of the crystal and its dissolution, λ , u . In this case, all the effects can be taken into consideration by the real growth rate of the crystal $\eta = \lambda - \mu$.

Thus, the complete system of equations has the form

$$\begin{aligned} \frac{\partial \rho_1}{\partial t} + \nabla (\rho_1 v_1) &= - \int_0^R \rho_2^0 f \eta dr, \quad \frac{\partial f}{\partial t} + \nabla (f v_2) + \frac{\partial}{\partial r} (f \eta) = 0, \\ \rho_1 \frac{d_1 v_1}{dt} &= - \alpha_1 \nabla p_1 + \nabla^k \tau_1^k - \int_0^R g f_{(12)} dr + \rho_1 F_1 - \int_0^R \rho_2^0 f \eta (v_2(r) - v_1) dr, \\ \frac{D_2}{Dt} v_2(r) &= - \frac{\nabla p_1}{\rho_2^0} + f_{(12)}(r) + F_2(r), \\ \rho_1 \frac{d_1 u_1}{dt} &= \tau_1^{kl} e_1^{kl} + \int_0^R g f_{(12)} (v_1 - v_2) dr - \\ &- \int_0^R \rho_2^0 f \eta \frac{(v_2 - v_1)^2}{2} dr - \int_0^R q_{1\sigma}(r) dr + \nabla q_1 + \rho_1 Q_1, \\ g \frac{D_2 u_2}{Dt} &= - q_{2\sigma}(r) + g Q_2(r), \quad f \frac{D_2 u_1}{Dt} = q_{1\sigma}(r) + q_{2\sigma}(r) - \rho_2^0 f \eta (i_2 - i_1), \\ [\rho_1^0(c) &= (\rho_1^0)_{c=0} / (1 - c), \quad \alpha_1 + \int_0^R r f dr = 1, \\ \rho_1 &= \rho_1^0 \alpha_1, \quad g = \rho_2^0 f r, \quad \rho_2^0 = \text{const}, \\ \frac{d_1}{dt} &= \frac{\partial}{\partial t} + v_1^k \frac{\partial}{\partial x^k}, \quad \frac{D_2}{Dt} = \frac{\partial}{\partial t} + v_2^k \frac{\partial}{\partial x^k} + \eta \frac{\partial}{\partial r}. \end{aligned}$$

With $n = 1$

$$\begin{aligned} \rho_2^0 \eta &= \frac{\beta \beta_c}{\beta + \beta_c} F_s (c - c_e(r)), \\ c_e &= c_e(T_1); \quad \beta_c = \beta_c(T_2(r), T_\sigma(r)). \end{aligned}$$

The value of β can be determined from the equation

$$Nu = A Pr Re_{(12)},$$

where

$$Re_{(12)} = (|v_1 - v_2|/v_1) a_1; \quad Pr = v_1/D_1; \quad Nu = \beta a_1/D_1;$$

v_1 is the viscosity of the solution; D_1 is the diffusion coefficient in the carrier phase; $q_{1\sigma} = 4\pi a^2 f \beta_1 (T_1 - T_\sigma)$; β_1 is the coefficient of heat transfer from the phase interface to the i -th phase; $q_1 = -\lambda_2 \nabla T_1$; $i_1 = \frac{c}{m} \Delta h + \left(1 - \frac{c}{m}\right) i_w + \frac{c}{m} i_{(21)}$ (according to [6]); $\Delta h = \Delta h(T_1, c)$ is the heat of solution at the saturation temperature; $i_w = \Delta H_{w(298)} + \int_{298}^{T_1} c_w dT_1$; $i_{(21)} =$

$\Delta H_{2(298)} + \int_{298}^{T_1} c_2 dT_1$; $i_2 = \Delta H_{2(298)} + \int_{298}^{T_2} c_2 dT_2$; $\Delta H_{W(298)}$, $\Delta H_{2(298)}$ are the standard enthalpies of water and the substance; c_w , c_2 are the specific heat capacities of water and the substance of the particles, respectively.

If we neglect the quantity $D_2 u_\sigma^1 / Dt$, then, the equation of the influx of heat to the phase interface leads to a finite algebraic equation for T_σ

$$4\pi a^2 f [\beta_1 (T_1 - T_\sigma) + \beta_2 (T_2 - T_\sigma)] - \rho_2^0 f \eta (i_2 - i_1) = 0.$$

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FLOW OF GENERALIZED NEWTONIAN AND BINGHAM LIQUIDS IN AN ANNULAR CAPILLARY

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The laws of motion of Newtonian [1, 2], viscoplastic [3-5], and non-Newtonian [6] liquids in an annular capillary (in the gap between two coaxial cylindrical tubes) have already been obtained. In the present paper we solve the problem of established horizontal flow of generalized Newtonian and Bingham liquids [7-9] in an annular capillary.

Let R_1 and R_2 be the internal and external radii, respectively, of the tubes forming the annular capillary, and r the radial cylindrical coordinate of a liquid particle in the flow cross section.

The flow of a generalized Newtonian liquid in a capillary under the action of a hydraulic pressure gradient I proceeds within the expanding ring $r_1 \leq r \leq r_2$ in such a way that the velocity $v(r)$ at some intermediate $r = r_0$ is a maximum and, decreasing nonsymmetrically in the direction of the walls, is a minimum at $r = r_1$ and $r = r_2$. We can accordingly distinguish two flow zones with different velocity laws $v_j(r)$ in the flow cross section. In the first zone ($j = 1$, $r_1 \leq r \leq r_0$) the velocity gradient $dv_1(r)/dr \geq 0$ and in the second zone ($j = 2$, $r_0 \leq r \leq r_2$) $dv_2/dr \leq 0$.

Considering the balance of the forces applied to an elementary annular layer of liquid in each zone we have

$$d\tau_j(r)/dr + \tau_j(r)/r = (-1)^j \rho g I, \quad (1)$$

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